

## COMPLEX MODEL OF THE EFFICIENCY OF RECTIFICATION PLATES.

### 3. COUNTERCURRENT MOTION OF PHASES IN PARTIAL MIXING OF A LIQUID

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*A model of mass transfer in the countercurrent motion and partial mixing of a liquid on a plate is analyzed. The liquid is considered to be consisting of two parts, one of which ( $\varphi$ ) is totally mixed and the other of which  $(1 - \varphi)$  moves in the regime of ideal displacement. The intensity of the mixing is determined by the relation of these parts. In the model, the compositions of the flows on ideal and actual plates at a certain distance  $h$  for the vapor and  $h_1$  for the liquid from the site of injection of the phases coincide; the vapor leaving an ideal plate and the liquid arriving at it are in equilibrium. The dependences of the difference of the concentrations of a highly volatile component in the liquid before and after the plate on the efficiency, the flow rates of the vapor and the liquid, and the coefficient of equilibrium are derived. Particular cases where  $h = h_1$  and  $h = h_1 = 0.5$  and where they are equal to zero or to unity are analyzed. Corresponding relations of the efficiencies are derived. The complex countercurrent model is analyzed with allowance for the mixing and for the fields of its application.*

A homogeneous composition of the phases, for which there are no internal conditions for the mixing of the liquid, is assumed in the countercurrent motion of a vapor and a liquid. In actual fact, the vapor rises in the form of bubbles of appreciable sizes and in the form of jets that have a substantial effect on the liquid moving toward them. The velocity of motion of liquid layers that are in contact with the vapor bubbles slows down, but for more peripheral layers it, conversely, increases. A local mixing of the liquid is observed on the step of contact. A change in the velocity of motion of considerable volumes of the liquid, the vapor fraction in which differs from the average value on the step, is also possible. In jet motion, the liquid is entrained by the vapor and is brought back to the site of its injection. On the step of contact, we observe an intense mixing of considerable volumes of the liquid.

The degree of mixing of a liquid is represented in different models in various ways [1, 2]. In the cell model, the mixing of a liquid is determined by the number of cells of total mixing between which the liquid moves in the regime of ideal displacement. In the circulation model, part of a liquid from the outlet of the plate comes back to its inlet. In the diffusion model, the degree of mixing is characterized by a dimensionless Péclet number.

The mixing of a liquid in the direction of its motion has a significant effect on the intensity of mass transfer (Fig. 1). This effect is less substantial in the perpendicular direction; therefore, in the horizontal layers the composition of the liquid is assumed to be homogeneous.

In the total mixing of a liquid, the vapor leaving the plate is in equilibrium with the discharging liquid [3]:

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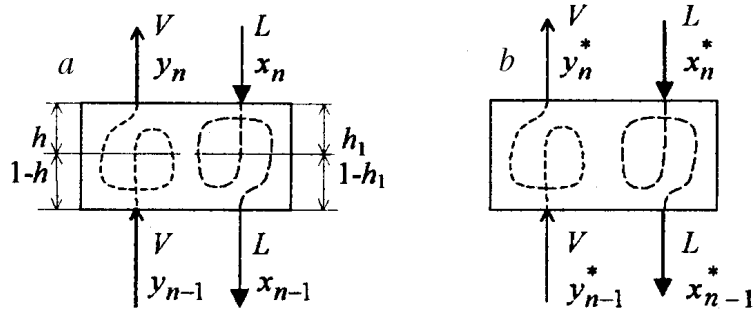


Fig. 1. Scheme of the countercurrent flows of the vapor and liquid phases on real (a) and ideal (b) plates in mixing.

$$(y_n^*)_{\text{con}} = mx_{n-1}^*, \quad (1)$$

while in the absence of any mixing we observe an ideal displacement of the liquid, and the vapor is in equilibrium with a liquid that arrives at the plate [4]:

$$(y_n^*)_{\text{dis}} = mx_n^*. \quad (2)$$

In [5], the mixing of a liquid in the cross motion of the phases is taken into account by the fact that part of the liquid is represented as totally mixed and the other part moves in the regime of ideal displacement. In the same manner, we can take into account the mixing of a liquid in its countercurrent motion with a vapor.

Let us assume that part of a liquid on a plate is totally mixed, and its other part moves in the regime of ideal displacement. We denote by  $\varphi$  the part of the liquid that is totally mixed on an ideal plate and that is characterized by the equilibrium condition (1). Then the amount of the liquid that moves in the regime of ideal displacement and for which condition (2) holds will be equal to  $1 - \varphi$ . In the total mixing of the entire liquid,  $\varphi = 1$ ; in ideal displacement,  $\varphi = 0$ . In this case, the composition of the vapor after an ideal plate as a function of the amount of mixed liquid can be represented in the form

$$y_n^* = \varphi (y_n^*)_{\text{con}} + (1 - \varphi) (y_n^*)_{\text{dis}} = \varphi mx_{n-1}^* + (1 - \varphi) mx_n^*. \quad (3)$$

The equations of material balance of a highly volatile component for ideal and real plates and the expressions of the efficiency in the vapor and liquid phases have the same form as in the concurrent motion of phases [3].

In analyzing the efficiency in the vapor phase, the content of the highly volatile component before and after an ideal plate is expressed by the same dependences as in the concurrent flow [3]:

$$y_{n-1}^* = hy_n + (1 - h) y_{n-1} - h \frac{y_n - y_{n-1}}{E_v}; \quad (4)$$

$$y_n^* = hy_n + (1 - h) y_{n-1} + (1 - h) \frac{y_n - y_{n-1}}{E_v}, \quad (5)$$

and the difference of its concentrations in the liquid before and after this plate is modified with account for (3) and (5):

$$x_n^* - x_{n-1}^* = \frac{1-h_1}{\varphi-h_1} x_n + \frac{h_1}{\varphi-h_1} x_{n-1} - \frac{h}{\varphi-h_1} \frac{y_n}{m} - \frac{1-h}{\varphi-h_1} \frac{y_{n-1}}{m} - \frac{1-h}{\varphi-h_1} \frac{y_n - y_{n-1}}{mE_v}. \quad (6)$$

In simultaneous solution of (4)–(6) and the equations of material balance of ideal and real plates [3], we obtain

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_v}{L(1-E_v) \left(\frac{1-h}{mV} + \frac{1-h_1}{L}\right) + \frac{L}{mV} E_v - 1 + \varphi}. \quad (7)$$

The same expression is derived when the efficiency in the liquid is used, which confirms the equality of the efficiencies in the vapor and liquid phases for the considered model of mass transfer, too, i.e.,

$$E_v = E_{\text{liq}} = E_{g,\varphi}. \quad (8)$$

If  $h = h_1$ , formula (7) takes the form

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{g,\varphi}}{(1-E_{g,\varphi})(1-h) \left(\frac{L}{mV} + 1\right) + \frac{L}{mV} E_{g,\varphi} - 1 + \varphi}, \quad (9)$$

and for  $h = h_1 = 0.5$ ,

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) 2E_{g,\varphi,m}}{\left(\frac{L}{mV} - 1\right) (1 + E_{g,\varphi,m}) + 2\varphi}. \quad (10)$$

Boundary conditions for the considered variant of a countercurrent flow, just as in the absence of mixing, are the conditions of the relationship of ideal and real plates that are inherent in the Murphree and Hausen models [6–9], where the distances  $h$  and  $h_1$  become equal to zero or to unity. Dependence (7) accordingly takes the form:

for  $y_{n-1}^* = y_{n-1}$  and  $x_{n-1}^* = x_{n-1}$  ( $h = 0$ ,  $h_1 = 1$ )

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{g,\varphi 1}}{\frac{L}{mV} - 1 + \varphi}; \quad (11)$$

for  $y_n^* = y_n$  and  $x_n^* = x_n$  ( $h = 1$ ,  $h_1 = 0$ )

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{g,\varphi 2}}{\frac{L}{mV} E_{g,\varphi 2} - E_{g,\varphi 2} + \varphi}; \quad (12)$$

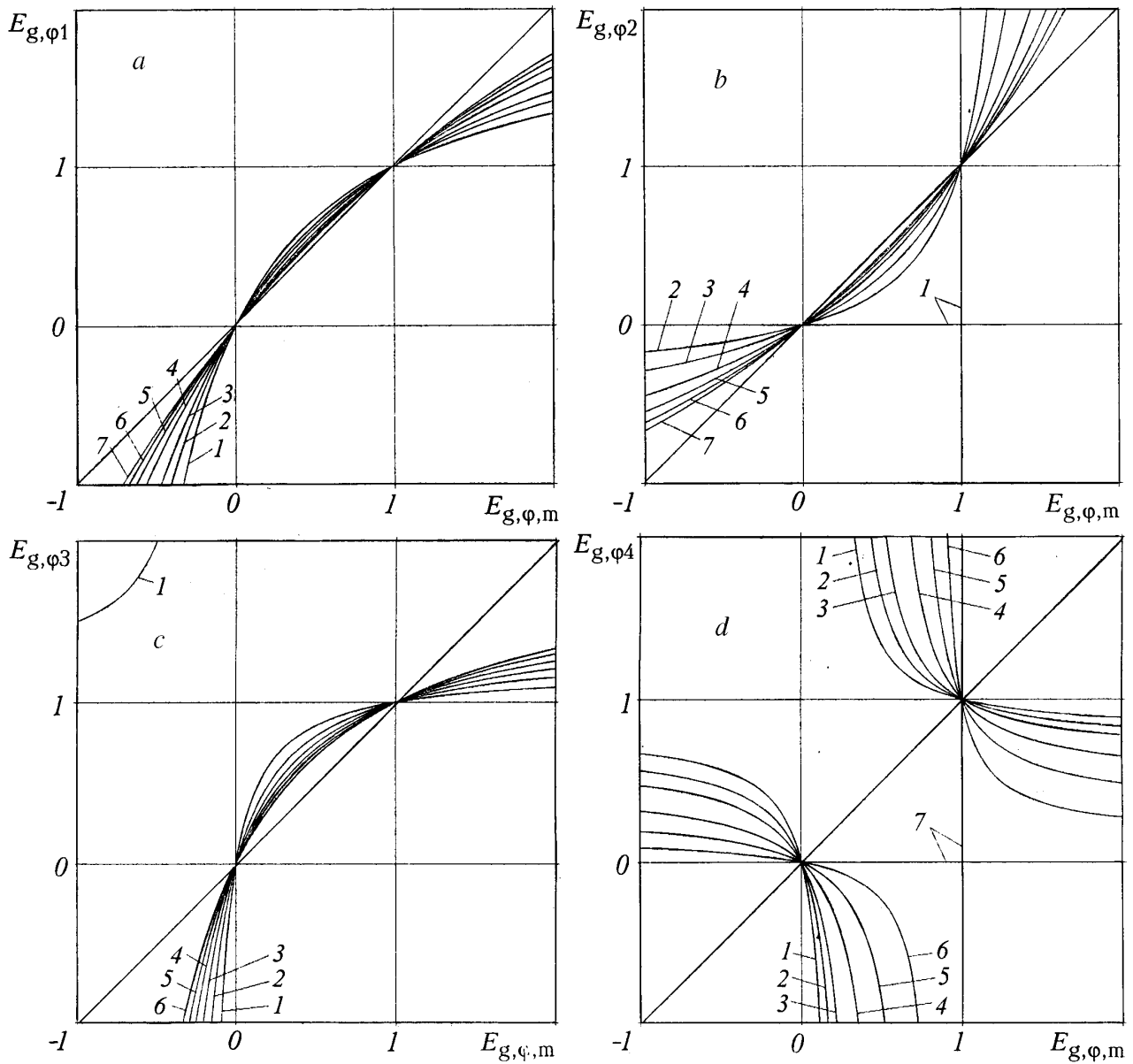


Fig. 2. Dependence of  $E_{g,\varphi1}$  (a),  $E_{g,\varphi2}$  (b),  $E_{g,\varphi3}$  (c), and  $E_{g,\varphi4}$  (d) on  $E_{g,\varphi,m}$  for  $L/mV = 1.5$  and different values of  $\varphi$ : a and b) 1) 0; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.6; 6) 0.8; 7) 1.0; c) 0; 2) 0.2; 3) 0.4; 4) 0.6; 5) 0.8; 6) 1.0; d) 1) 0; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.5; 6) 0.8; 7) 1.0.

for  $y_{n-1}^* = y_{n-1}$  and  $x_n^* = x_n$  ( $h = h_1 = 0$ )

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{g,\varphi3}}{\frac{L}{mV} - E_{g,\varphi3} + \varphi}; \quad (13)$$

for  $y_n^* = y_n$  and  $x_{n-1}^* = x_{n-1}$  ( $h = h_1 = 1$ )

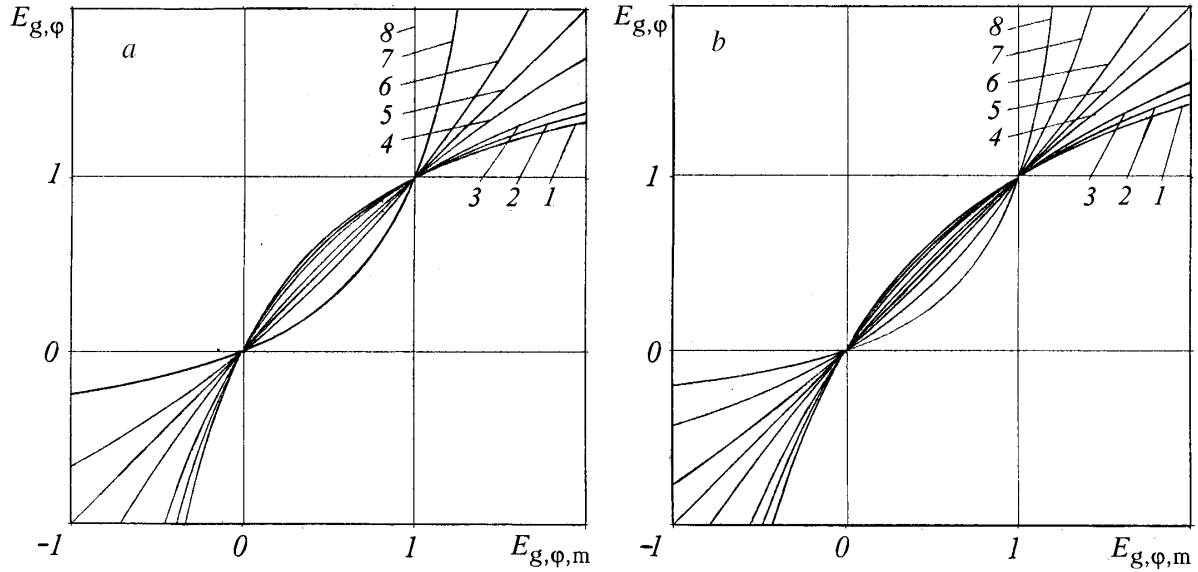


Fig. 3. Dependence of  $E_{g,\phi}$  on  $E_{g,\phi,m}$  for  $L/mV = 1.5$ ,  $\phi = 0.5$ ,  $h_1 = 0.5$  (a), and  $h = 0.5$  (b) and for different values of  $h$  (a) and  $h_1$  (b): 1) 0; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.5; 6) 0.6; 7) 0.8; 8) 1.0.

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{g,\phi 4}}{\frac{L}{mV} E_{g,\phi 4} - 1 + \phi} \quad (14)$$

By equating the right-hand sides of formulas (10)–(14), we obtain the relation of the efficiencies

$$\frac{\frac{L}{mV} - 1 + 2\phi}{2E_{g,\phi,m}} + \frac{L}{2mV} - \frac{1}{2} = \frac{\frac{L}{mV} - 1 + \phi}{E_{g,\phi 1}} = \frac{L}{mV} - 1 + \frac{\phi}{E_{g,\phi 2}} = \frac{\frac{L}{mV} + \phi}{E_{g,\phi 3}} - 1 = \frac{L}{mV} - \frac{1 - \phi}{E_{g,\phi 4}} \quad (15)$$

From (9) or (10) we derive the following formulas for calculating the efficiency of a real plate in generalized form and in a particular case, respectively, for  $h = h_1 = 0.5$ :

$$E_{g,\phi} = \left[ (1-h) \frac{L}{mV} + \phi - h_1 \right] \Bigg/ \left( \frac{x_n - \frac{y_{n-1}}{m}}{x_n - x_{n-1}} - h \frac{L}{mV} - h_1 \right) \quad (16)$$

$$E_{g,\phi,m} = \left( \frac{L}{mV} + 2\phi - 1 \right) \Bigg/ \left( \frac{x_n + x_{n-1} - 2 \frac{y_{n-1}}{m}}{x_n - x_{n-1}} - \frac{L}{mV} \right) \quad (17)$$

Figure 2 gives graphic representations of the dependences of individual pairs of efficiencies of the complex model of mass transfer in countercurrent phase motion in accordance with formula (15).

Mixing decreases the efficiency of the countercurrent flow for  $h = 0$  (Fig. 2a and c), improves it for  $h = 1$  (Fig. 2b), and in all cases make the values of the pairs of efficiencies closer, although they do not become equal. The quantity  $\phi$  has the minimum effect when  $h = 0$  and  $h_1 = 1$  (Fig. 2a) and the maximum

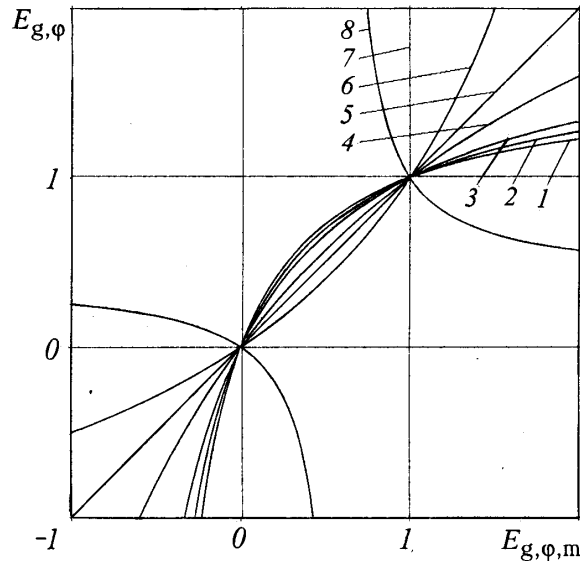


Fig. 4. Dependence of  $E_{g,\varphi}$  on  $E_{g,\varphi,m}$  for  $L/mV = 1.5$ ,  $\varphi = 0.5$ , and different values of  $h = h_1$ : 1–8) notation is the same as in Fig. 3.

effect when  $h = 1$  and  $h_1 = 0$  (Fig. 2b). The limiting case where  $h = h_1 = 1$  (Fig. 2d) is unacceptable for an analysis of the operation of the real plate since  $E_{g,\varphi}$  cannot be higher than unity or lower than zero, and in this variant the real values of one efficiency correspond to the unreal values of the other.

The mixing of the liquid also has an effect on the character of the relation of the efficiencies with a fixed value of one distance  $h$  or  $h_1$  (Fig. 3) or with the identical change in them (Fig. 4). As compared to the ideal displacement in countercurrent phase motion [4] it decreases the range of possible changes in the efficiencies. In particular, when  $h_1 = 0.5$  (Fig. 3a) or  $h = 0.5$  (Fig. 3b) there are no unreal values of one efficiency for the real values of the other, whereas in the absence of mixing these features occur for, respectively,  $h < 0.666$  and  $h_1 < 0.75$ . A similar regularity is also observed with the identical change in  $h$  and  $h_1$ . The character of the dependence of the efficiency of mass transfer on  $h$  or  $h_1$  remains constant; it decreases with increase in these distances. Therefore, in mixing of the liquid, it is appropriate to increase  $h$  and  $h_1$  with the aim of decreasing the calculated efficiency.

Thus, the complex model of mass transfer extends the field of application of the known models and supplements them with the features of partial mixing of a liquid.

## NOTATION

$E$ , efficiency of the plate;  $\varphi$ , amount of the totally mixed liquid on the plate;  $h$  and  $h_1$ , dimensionless distance from the site of injection of the vapor and the liquid, respectively, to the surface of equality of the concentrations of the phases in the ideal and actual plates;  $L$ , molar flow of the liquid;  $m$ , coefficient of equilibrium;  $V$ , molar flow of the vapor;  $x$  and  $y$ , concentration of the highly volatile component in the liquid and the vapor, respectively. Subscripts: g, countercurrent motion of the phases;  $\delta$ ,  $\varphi$ , m, countercurrent motion of the phases in mixing and for  $h = h_1 = 0.5$ ;  $\varphi$ , allowance for the mixing of the liquid; liq, liquid phase;  $n$ , number of the considered plate;  $n-1$ , number of the preceding plate in the direction of vapor motion;  $v$ , vapor phase. Superscript: \*, ideal conditions.

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